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On the Noise Parameters of Isolator and Receiver with Isolator at the Input

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***Abstract*—Noise parameters of an isolator and those of a receiver with an isolator at the input are reviewed. Some comments on recently published results are offered.**

I. Introduction

Isolators are very commonly used in low-noise receivers as well as in noise measuring systems (for instance, [1]–[5]). Usually their purpose is to isolate either the noise source or the receiver from the rest of the system. In these cases, the noise properties of either the isolator alone or the receiver with the isolator at the input need to be known. This paper offers a brief discussion of the noise properties of these two-ports and gives closed-form expressions in some idealized cases for the set of noise parameters, namely minimum noise temperature T_{min}, optimum source reflection coefficient Γ_{opt}, and noise parameter N as defined in [9]. A short discussion of some of the recently published results [4], [5], [11], [15] is also given.

II. Theory

Consider a linear, noisy system schematically presented in Fig. 1. Signal parameters of both an isolator and a receiver are represented by chain matrices $[A_I]$ and $[A_R]$ and their noise parameters by correlation matrices $[C_{AI}]$ and $[C_{AR}]$, respectively [6]. An isolator is a passive, nonreciprocal, linear two-port with thermal noise generators only and, therefore, its noise parameters can be derived from its signal parameters [7]. The appropriate

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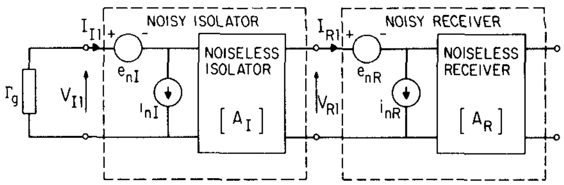

$$[C_{AI}] = \begin{bmatrix} \overline{e_{nI}^2} & \overline{e_{nI}\, i_{nI}^*} \\ \overline{e_{nI}^*\, i_{nI}} & \overline{i_{nI}^2} \end{bmatrix} \qquad [C_{AR}] = \begin{bmatrix} \overline{e_{nR}^2} & \overline{e_{nR}\, i_{nR}^*} \\ \overline{e_{nR}^*\, i_{nR}} & \overline{i_{nR}^2} \end{bmatrix}$$

Fig. 1. A cascade connection of isolator and receiver.

equivalent networks with pertinent formulas [7], [8] are given in Fig. 2. Then the correlation matrix $[C_A]$ completely characterizing the noise parameters of the system at the input port of the isolator is [8]

$$[C_A] = [C_{AI}] + [A_I][C_{AR}][A_I]^\dagger \tag{1}$$

where the "dagger" designates the complex conjugate of the transpose of $[A_I]$ matrix. Any desired set of noise parameters can be derived from $[C_A]$ (for instance, [6], [8]–[10]).

It should be stressed that this approach is not limited by the particular realization of an isolator as, for instance, a Faraday rotation isolator or an isolator made of a circulator with one port terminated. The noise properties of both isolators are the same if they are at the same physical temperature and their two-port signal parameters are the same.

Although the formulas presented in Figs. 1 and 2 and also (1) lend themselves easily to computer implementation (for instance, [13], [14]), and, therefore, are convenient to use in computer-aided design and/or computer-aided measurement, it is very instructive to discuss the conventional noise parameters of an ideal isolator, which is equivalent to an ideal circulator with one port terminated (Fig. 3(a)). It follows directly from Twiss's [7] general approach or from simple physical reasoning that the noise parameters of an ideal isolator are

$$T_{min} = 0, \quad \Gamma_{opt} = 0, \quad N = \frac{T_a}{4T_0} \tag{2}$$

where

T_{min}	minimum noise temperature,
Γ_{opt}	optimum reflection coefficient of the source,
$T_0 = 290$ K	standard temperature,
T_a	physical temperature of a circulator termination, (or physical temperature of an isolator),
N	noise parameter defined in [9].

It is instructive to give physical interpretation of the noise parameters given by (2). An ideal isolator emits a noise wave from its input port, which is totally absorbed by the source if $\Gamma_g = \Gamma_{opt} = 0$. In this case, no noise generated by the isolator appears at its output and $T_{min} = 0$. If $\Gamma_g \neq 0$, part of the noise is reflected back and appears at the isolator output, which gives rise to parameter $N > 0$.

Small losses L of an isolator in the forward direction can be modeled accurately by a cascade connection of an ideal isolator and a matched attenuator, as shown in Fig. 3(b). In this case of a slightly lossy isolator, the noise parameters are

$$T_{min} = T_a(L-1) \quad \Gamma_{opt=0} \quad N = \frac{T_a + T_{min}}{4T_0}. \tag{3}$$

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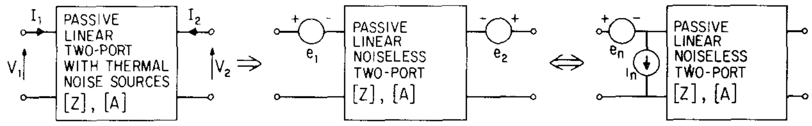

$$[Z] = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \qquad [C_Z] = \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} \times [e_1^* \; e_2^*] = \qquad [C_A] = [T] \times [C_Z] \times [T]^\dagger$$

$$[A] = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \qquad = 4kT_a\Delta f \begin{bmatrix} \mathrm{Re}\, Z_{11} & \frac{1}{2}(Z_{12}+Z_{21}^*) \\ \frac{1}{2}(Z_{12}^*+Z_{21}) & \mathrm{Re}\, Z_{22} \end{bmatrix} \qquad [T] = \begin{bmatrix} 1 & -A \\ 0 & -C \end{bmatrix}$$

Fig. 2 An isolator as a passive, nonreciprocal, linear two-port with thermal noise sources only The formulas given are from [7] and [8]

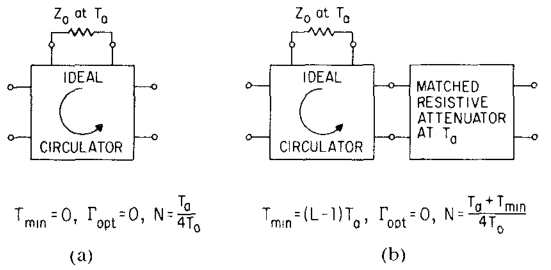

Fig. 3 (a) An ideal isolator, as an ideal circulator with one port terminated and its noise parameters. (b) An approximate model of slightly lossy isolator and its noise parameters.

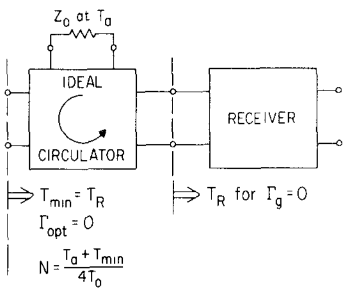

Fig 4 An approximate model of a cascade connection of isolator and receiver and its noise parameters

If an isolator cannot be described by these simple models, its noise properties are best treated by the general approach outlined in Fig. 2.

Finally, if an ideal isolator is followed by a receiver as showed in Fig. 4 (small losses of an isolator can be modeled as part of a receiver), the noise parameters of this system at the input port of the isolator are

$$T_{\min} = T_R, \quad \Gamma_{\mathrm{opt}} = 0, \quad N = \frac{T_a + T_{\min}}{4T_0}. \tag{4}$$

Therefore, the noise temperature T_n of the system of Fig. 4 for arbitrary Γ_g is

$$T_n = T_{\min} + (T_a - T_{\min}) \frac{|\Gamma_g|^2}{1 + |\Gamma_g|^2}. \tag{5}$$

It is clear from (5) why only the magnitude of the source reflection coefficient needs to be known and also why it is advantageous to keep the termination of a circulator cold.

If the simple model of Fig. 4 does not apply, the use of (1) is recommended, which requires the knowledge of all four noise parameters of a receiver, two-port signal parameters of an isolator, and its physical temperature.

III. Comments

The equivalence of noise behavior between a Faraday rotation isolator and an isolator made of a circulator with one port terminated has been discussed in a recent paper [11], where the approach presented by Siegman in an earlier work [12] has been reviewed. The conclusions of both papers [11], [12] on this subject follow directly from the much more general result of Twiss [7].

In [4], [5], and [15], expressions for the noise figure of a system with the isolator at the input for arbitrary Γ_g are given. It should be noted that these expressions are valid only if the physical temperature of the isolator T_a is equal to the standard temperature $T_0 = 290$ K. This condition was not clearly stated in [4], [5], and [15].

The formulas presented in this paper follow directly from results published many years ago [7], [8]. The author feels, however, in view of recently published [4], [5], [11], [15] that it is worthwhile to present these in the form congruent with that commonly used in the description of noisy two-ports.

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Two Core Radii For Minimum Total Dispersion In Single-Mode Step-Index Optical Fibers

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***Abstract*—Starting from the operating wavelength and the chemical composition of the materials that integrate the core and cladding of an optical fiber, a method was developed for the calculation of the values of the core radii; it allows fiber operation in a monomode region with minimum total dispersion.**

The study is restricted to step-index fibers and the selected theoretical model is based on the weakly-guiding characteristic equation. From these considerations it is possible to obtain two different values of core radii for a given source operating wavelength.

The theory described allows the characterization of an optical fiber for use with a given light source and extends a previously described theory.

I. Introduction

The spread of light pulses transmitted through single-mode optical fibers is caused by two main factors, material dispersion and waveguide dispersion. The first factor results from the dependence of the refractive indexes of the materials used in the construction of the core and cladding of the optical fiber on the wavelength. The second factor takes into consideration the effect of the geometry of the guiding structure (the optical waveguide) on its fundamental mode. Both factors combine and the result is known as the total dispersion. It is worth mentioning that this combination does not result from the simple addition of the two factors mentioned above but rather it is much more complex than this [1], [2].

In order to reduce as much as possible the pulse spread and obtain as a consequence an increase in the operating passband available, optical fibers with minimum total dispersion at the source wavelength, $\lambda = \hat{\lambda}$, should be used. The value of $\hat{\lambda}$ is obtained through the solution of the total dispersion equation.

Various methods have been proposed for solving this problem. In the case of single-mode step-index optical fibers these methods are essentially based on three procedures: 1) the use of the weakly guiding characteristic equation [1], [4], [5]; 2) the use of asymptotic approximations for the eigenvalues of the weakly guiding characteristic equation [6], [8], [9]; and 3) the use of the exact characteristic equation [3].

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For the calculation of the wavelength for minimum total dispersion $\hat{\lambda}$, using the exact characteristic equation, the complexity of the algorithms used and the large number of data to be manipulated [3] require computer systems of medium or large size. When the weakly guiding characteristic equation is used, the amount of data to be manipulated is reduced due to the relative simplicity of the equations and algorithms involved. When asymptotic approximations are used the computational procedures may be implemented on small programmable calculators. The use of any of these procedures will depend on the available computational system and the required precision of the results.

For the calculation of $\hat{\lambda}$ it is first necessary to have prior knowledge of the optical-fiber physical characteristics, such as the core radius and the chemical composition of the materials used for the construction of the fiber core and cladding. Once the value of $\hat{\lambda}$ is found, the most appropriate optical source to operate the fiber under minimum total dispersion may be chosen. This is an analysis procedure where, given an optical fiber, the optimum source to operate with the fiber can be found.

An opposite problem to the one just described, which consists of synthesizing an optical fiber for optimal operation with a given optical source, is usually of particular interest.

A method for accomplishing the synthesis of single-mode step-index optical fibers has been reported in a previous work [10]. Asymptotic approximations proposed by Miyagi and Nishida [8] were used in that work. The chemical composition of the core and cladding materials were assumed known and the available light source wavelength was chosen equal to $\hat{\lambda}$. In this case, the total dispersion equation is used for the calculation of the fiber core radius. From the characteristics of the method adopted, the calculated radius is the one that allows pulse transmission with minimum total dispersion when the fiber operates with the wavelength of the available source. Due to its simplicity, all the computational procedures were implemented on a small programmable calculator. However, due to the asymptotic approximation used, only one value of radius for minimum total dispersion was found. The existence of two core radii for the same value of $\hat{\lambda}$ was suggested in a previous work [3].

In the present work the weakly guiding characteristic equation was used as the theoretical basis for the synthesis of single-mode step-index optical fiber. The use of this equation allows for the calculation of the two-core radii that yield minimum total dispersion, as predicted in [3]. As expected, the use of the weakly guiding characteristic equation reduces the amount of data to be manipulated and the complexity of the algorithms to be adopted.

In the following sections, a description of the theory is given along with the results of a few cases.

II. Basic Equations

The total dispersion equation which is the wavelength derivative of the transit time per unit fiber length, is given by [6]

$$D_T = (\lambda / c n_e)\{(1-b)\nu_2 + b\nu_1 + 2b'\phi + b''\theta/2 - (n_e)^{-2}[n_2 n_2' + b\phi + b'\theta/2]^2\} \quad (1)$$

where c and λ are the free-space phase velocity and wavelength of the light wave, respectively; b is the normalized propagation constant given by the relation

$$b = 1 - \frac{U^2}{V^2} = \frac{W^2}{V^2} \quad (2)$$

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